A. ONLINE APPENDIX: Formal Analysis of Leader Loyalty (Section 5.4)

A.1. Preliminaries

We consider an (infinitely) repeated game in which, in each period, there is a contest between two leaders. At the end of the period, a leader exits (and is replaced by a new leader) when either (1) his team wins, (2) his agent quits when bad news is revealed, or (3) the leader gives up after a loss.

As in our baseline model, in each period, both leaders chose the skill of their follower optimally, and have the choice between an H-agent and an L-agent (the extension to multiple followers per leader is direct).

We assume throughout that we are in the (interesting) case where the unique static equilibrium is the one where both leaders hire a low-skill agent L. The question we explore is whether and how loyal leaders allow for this equilibrium to be improved.

Assumption 4. (Low skill agents in static equilibrium) -$\delta < S^*$

Just as the follower, the leader is motivated by the value of the prize, W, but may also have something better to do than wait around for another chance to win. In particular, after any loss, the leader has the opportunity to run again. However, with probability φ, he has an outside option $\omega_L > W/2$ and will 'quit'.

A.2. The value of a loyal leader

Let ρ be the discount factor, that is how much followers value the future. If a leader who runs again always rehires a loyal follower (but not a disloyal one), then the expected utility of an H follower who remains loyal and does not quit after bad news is:

$$U_F^{stay} = \frac{1}{2}\left[P(win|\delta)W + P(lose|-\delta)\varphi\rho U_F^{stay}\right] + \frac{1}{2}\left[P(win|-\delta)W + P(lose|\delta)\varphi\rho U_F^{stay}\right]$$

As in the static equilibrium, it will be sufficient to show that given an opponent team with an L agent, it is optimal to hire an H agent. When facing an L-agent, we have that

$$U_F^{stay} = \frac{W}{2} \frac{G(H-L-\delta) + G(H-L+\delta)}{\left(1 - \frac{\varphi\rho}{2}\left[1 - G(H-L-\delta) + 1 - G(H-L+\delta)\right]\right)} \tag{31}$$

Following bad interim news, an H agent facing an L agent then stays if and only if

$$G(H-L-\delta)W + (1 - G(H-L-\delta))\,\varphi\rho U_F^{stay} \geq \omega \tag{32}$$

Note that U_F^{stay} is increasing in $\varphi\rho$ and $U_F^{stay} = W$ if $\varphi\rho = 1$. Hence, (32) is always satisfied if $\varphi\rho = 1$ and the leader always runs again/there is no discounting. In contrast, since -$\delta < S^*$, (32) is violated if $\varphi\rho = 0$ and the leader never runs again. Since the LHS of (32) is increasing in $\varphi\rho$, it follows that there exists a $\rho^r \in (0, 1)$ such that the incentive constraint (32) is satisfied if and only if

$$\varphi\rho \geq \rho^r.$$

We obtain the following characterization of the equilibrium, operating analogously to Proposition 1 :

Proposition 11. *For any $-\delta < S^*$, there exists a $\rho^r \in (0, 1)$ such that:*

(i) If $\varphi\rho \geq \rho^r$, the only equilibrium team composition is "meritocratic": both leaders choose an H follower;

(ii) If $\varphi\rho < \rho^r$, the only equilibrium has both leaders choosing a L follower (as in the static case).

Proof of Proposition 11. If $\varphi\rho \geq \rho^r$, a leader facing an opponent with an L agent, will choose an H agent, and will win with a probability larger than 1/2 as the H agent stays with bad news. A leader facing an opponent with an H agent then also prefers to hire an H agent, giving him a probability of winning equal to 1/2. If, instead, he were to hire an L agent, his opponent would win with a probability larger than 1/2. Conversely, if $\varphi\rho < \rho^r$, a leader facing an opponent with an L agent, will also choose an L agent, giving him a probability of winning equal to 1/2. If, instead, he were to hire an H agent, the latter would quit with bad news, resulting in a probability of winning less than 1/2. For the same reason, hiring an L agent is then also optimal when the opponent hires an H agent, as this would give the latter a probability of winning greater than 1/2. □

From Proposition 11, leader loyalty may solve the problem of disloyal subordinates: if leaders are sufficiently loyal (likely to run again) and followers are sufficiently patient, high-skill followers will be loyal and stay even in the presence of bad news.

Note finally that as in our static model, strategic considerations matter in talent choice. Condition (32) only guarantees that a meritocratic H agent stays loyal when facing a team with a low-skilled opponent. It is therefore possible that, when both teams choose H followers, the H follower receiving bad interim news gives up. Nevertheless, choosing a meritocratic H follower is still optimal in the latter case because of the 'discouragement effect': an H follower induces the rival team to give up when they face setbacks, whereas a loyal but low-skill follower would not have this strategic effect.

A.3. The cost of engendering follower loyalty

In the analysis above, we have posited that there is an exogenous probability that leaders will run again after a loss and rehire an agent. We now explore the determinants of such a decision. In particular, "loyal leaders" are not simply leaders that "run again" after a loss. Loyal leaders, by sticking with their followers even when they are not suitable to the particular situation or task, engender in turn loyalty from their followers.

Model amendments. To explore this type of loyalty, we amend our model as follows:

Agent types. There are no intrinsic types; instead agents can be suitable for one project and unsuitable for the next. In particular, each period, a given agent can be H or L. The suitability of the agent evolves period by period according to a Markov process. An H agent becomes L with probability $\lambda > 0$, and an L agent always stays L.

Outside options. If an agent does not quit, she stays, and depending on the result she obtains (as previously) either W (when the tournament is won) or 0. The agent, in each period, also has access to an outside option ω if and only if they are an H in that period. For simplicity, once a worker quits or is not rehired by her current leader, she never gets rehired anymore.

Timing. We study the incentives of the leader at time t to stick with the agent from time $t-1$, even when the agent's type for period t equals L. (i) At the start of each period, the new type of the follower is publicly observed. (ii) Then, a leader who lost in the previous period decides whether to keep the same follower or choose a new one. Each period, there is a continuum of agents to choose from, so there is always an H agent available. (iii) Finally, the tournament takes place. A leader (and his follower) who wins, exits the game, and is replaced by a new leader in the next period.

Equilibria with leader loyalty We now explore the existence of equilibria where new leaders select an H follower in the first period, and subsequently stay loyal to this follower, even when she turns mediocre (becomes an L type). While a long-term relationship then improves the ability of a leader to motivate high-skilled agents to stick around, ultimately, he is also unwilling to get rid of that agent when unavoidably, the latter becomes less suited for the job.

As in the previous section, whenever recruiting an H-agent is optimal when the opponent leader recruits an L-agent, recruiting an L-agent is necessarily a dominated strategy when facing an H-agent as opponent.

Assume therefore that the competing leader has recruited an L agent and denote by U_F^H the continuation utility, at the start of a new period, of an H-agent who never quits following bad interim news. We formally derive U_F^H in the proof of Proposition 12.

Following bad interim news, an H agent facing an L agent then stays if and only if

$$G(H - L - \delta)W + (1 - G(H - L - \delta))\,\varphi\rho U_F^H \geq \omega \tag{33}$$

Analogous to the previous section (see Proof below), one can show that there exists a $\rho^R \in (0,1)$ such that the incentive constraint (33) is satisfied if and only if

$$\varphi\rho \geq \rho^R.$$

This yields the following proposition:

Proposition 12. *1. For any $-\delta < S^*$, there exists a $\rho^R \in (0,1)$ such that an H agent facing an L agent stays loyal after bad news if and only if*

$$\varphi\rho > \rho^R$$

2. If $\varphi\rho > \rho^R$, the leader initially starts off with an H-follower (a "meritocratic" team) but he stays loyal to this follower when she turns mediocre.

Proof of Proposition 12. The utility of a follower who does not quit when observing bad news is given recursively by (depending on his type in a given period):

$$U_F^H = \frac{W}{2}\left[P^H(win|\delta) + P^H(win|-\delta)\right] + \frac{\rho\varphi}{2}\left[P^H(lose|\delta) + P^H(lose|-\delta)\right]$$

$$\times\left[(1-\lambda)U_F^H + \lambda U_F^L\right]$$

$$U_F^L = \frac{W}{2}\left[P^L(win|\delta) + P^L(win|-\delta)\right] + \frac{\rho\varphi}{2}\left[P^L(lose|\delta) + P^L(lose|-\delta)\right]U_F^L$$

where $P^H(win|\delta)$ and $P^L(win|\delta)$ are the probabilities of winning given good interim news when the followers are, respectively, of type H and type L (and the opposing team consists of L agents). It follows that

$$U_F^L = \frac{W}{2}\frac{\left(P^L(win|\delta) + P^L(win|-\delta)\right)}{\left(1 - \frac{\varphi\rho}{2}\left(P^L(lose|\delta) + P^L(lose|-\delta)\right)\right)},$$

from which

$$U_F^H = \frac{W}{2}\frac{\left(P^H(win|\delta) + P^H(win|-\delta)\right)}{\left(1 - \frac{(1-\lambda)\varphi\rho}{2}\left(P^H(lose|\delta) + P^H(lose|-\delta)\right)\right)}$$

$$+\frac{\lambda\rho\varphi}{2}\frac{\left(P^L(lose|\delta) + P^L(lose|-\delta)\right)}{\left(1 - \frac{(1-\lambda)\varphi\rho}{2}\left(P^H(lose|\delta) + P^H(lose|-\delta)\right)\right)}\left(U_F^L\right)$$

which is increasing $\varphi\rho$. Moreover, for $\varphi\rho = 1$, both $U_F^L = W$ and $U_F^H = W$.

Consider now incentive constraint (33). Since $U_F^H = W$ for $\varphi\rho = 1$, (33) is always satisfied if $\varphi\rho = 1$. In contrast, since -$\delta < S^*$, (33) is violated if $\varphi\rho = 0$ and the leader never runs again. Since the LHS of (33) is increasing in $\varphi\rho$, it follows that there exists a $\rho^R \in (0,1)$ such that the incentive constraint (33) is satisfied if and only if

$$\varphi\rho \geq \rho^R.$$

The remainder of the proof is analogous to that of Proposition 11.